

Computational design and fabrication of highly customizable architectural space frames: Making a flat-cut Weaire-Phelan structure

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Abstract

This paper documents a computational approach to the design, fabrication, and assembly of customizable space structures built entirely out of flat-cut interlocking elements without the need of nodes, fasteners, cement, or glue. Following a Research by Design (RbD) methodology, we establish a framework comprising geometric and parametric modeling, structural analysis, and digital fabrication stages to examine the following research question: how might the modularity of a construction kit be combined with the plasticity of parametric descriptions to facilitate the design and fabrication of flat-cut space structures? We find that an adaptive joint design that resolves local deformations at the node and element levels can facilitate the construction of flat-cut space structures by making modular components responsive to local geometric, material, and mechanical demands. The research centers on the design and construction of an architecture-scale installation based on the Weaire-Phelan structure—an aperiodic space-filling geometric structure that approximates the geometry of foam—entirely out of flat-cut interlocking elements. Documenting the process in technical detail, as well as some limitations, the paper contributes to recent efforts to develop digital materials suitable for architectural applications. In addition, it contributes to extend the formal and architectural possibilities of flat-cut space structure design by facilitating “bottom-up” design explorations in concert with the structure’s tectonic resolution.

Keywords

Computational design, generative fabrication, construction kit

Introduction

Research in computational design and adjacent fields has pondered the development of discrete design languages exploring, for example, pre-rational approaches to design modeling,¹ digital materials,² generative approaches to digital fabrication,³ and self-assembly.⁴ Common to these efforts is an interest in

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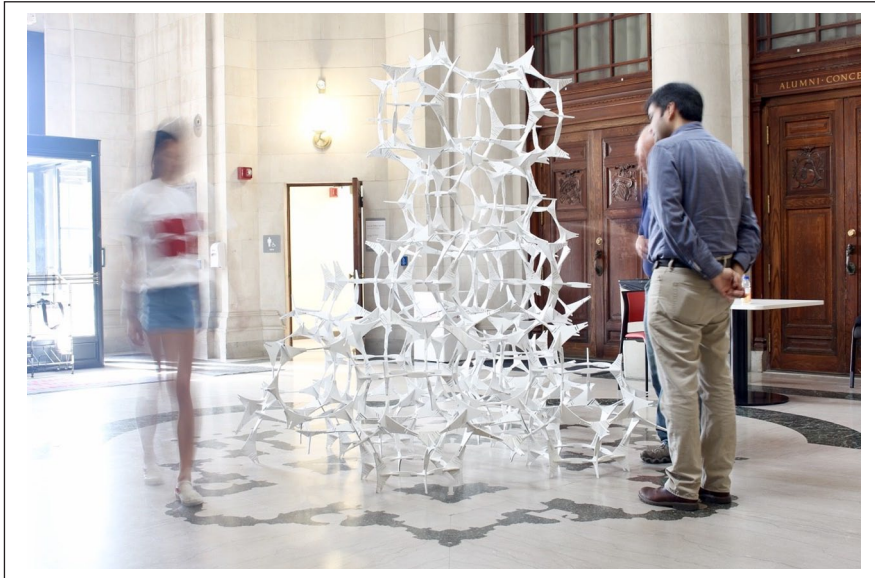


Figure 1. The Weaire-Phelan polyhedral structure assembled from discrete planar components (Photo by Chitika Vasudeva).

aligning geometric and material factors to simplify assembly, and in the potential of digital fabrication technology to facilitate part production. Put simply, digitally produced parts that account for a structure's assembly logic may afford design flexibility while helping simplify assembly. Extending these efforts this paper asks how parametric and modular logics may combine in the design and fabrication of spatial structures with flat-cut interlocking parts. To address this question we follow a Research by Design⁵ methodology to develop an approach to designing and fabricating complex polyhedral structures entirely out of dry mechanical assemblies suitable for rapid prototyping. We find that by combining the flexibility of parametric descriptions and the modular logic of a construction kit, this approach contributes to the literature on digital materials and extends the geometric and tectonic possibilities of digital design and construction. The paper documents the computational and material logic of the system, as well as a series of prototypes and a full-scale installation based on the Weaire-Phelan structure—a polyhedral structure that approximates the geometry of foam (Figure 1).

The paper further reflects on the significance of our method and its capacity to enable bottom-up design exploration. Potential applications in architecture and engineering are discussed.

Background

Computational design and fabrication methods have opened new aesthetic, tectonic, and configurational possibilities for interlocking, modular structures, or “kits.” These systems allow for reversible assembly and, depending on their specific design, can reduce the accumulation of dimensional errors along the construction process. This is illustrated by Neil Gershenfeld's “Great Invention Kit,” a system based on a single two-dimensional friction-fit module which has the potential to progressively self-correct for gravity and material deformation as more pieces are added.² In a similar vein, computational designer Gabe Johnson proposed a tool kit for designers to build physical structures with the goal of supporting creative design exploration while providing a way to learn about complex assemblies.⁶ Approaches to interacting with such

languages vary, including efforts to robotically assist building through discrete design elements, aiming at creating a more direct connection between design and fabrication,⁷ and to interactively assist the process of configuring designs. For example, Skouras et al. presented an interactive method for the quadrilateral interlocking system, which can compose a variety of complex forms using a flexible plastic plate. The forward design tool they developed allowed novice users to achieve a desired shape, and automatically-generated assembly instructions.⁸ Similarly, Weizmann et al. introduced a computational tool for generating interlocking components for architectural structures.⁹ Space structures, or space frames, offer an interesting challenge for this kind of construction. Common space structures are “bar and node” systems which involve highly processed materials and complex and labor-intensive assembly processes. Detachable connection mechanisms with removable connectors exist (see, e.g. Mero, Space Deck, Triodetic, and Unitrut systems). However, external connectors such as fasteners and bolts impose conditions to manufacture and assembly. Adhesives such as glue and cement are widely used to connect components in space structures, similarly welding¹⁰ can be used for connecting metal components in space structures. However, once assembled, space structures are usually difficult to separate and reuse. Instead of using non-reversible adhesives, Jenett et al. describes an approach for the discrete and reversible assembly of aileron system with modular building blocks joined by slot-type connections.¹¹

In particular, previous studies by the corresponding author on tectonic languages comprising reduced vocabularies of digitally-fabricated interlocking elements¹² are an important starting point of this research (See Figure 2). In these, the need to resolve friction-fit assemblies between non-orthogonal planes posed a tectonic challenge that we undertake here. These projects show that the potential of generative modular design logic lies in defining and exploring a “design space” that reflects the constraints and opportunities of fabrication technologies, and the opportunities afforded by materials’ own behavior and adaptability. In addition, these experiments suggest that relying on 2D manufacturing operations, and limiting material choices to planar surfaces, can yield important efficiencies which can be relevant to the construction of temporary structures in contexts where processed materials, tools, and/or labor are scarce.¹¹ Extending the lines of inquiry outlined above, in this paper we ask: how we may enrich the modular logic of a construction kit with the parametric plasticity of computational descriptions in order to facilitate the design, fabrication, assembly, and disassembly of architectural-scale space structures built entirely out of flat-cut interlocking parts?

Methods

Following a Research by design methodology (RbD), we aim at developing a computational approach to space structure design and construction relying on flat-cut interlocking parts that can be digitally fabricated and easily assembled. Research by design is “a type of academic investigation through which design is explored as a method of inquiry, by the development of a project and also exploring the different materials by which a design is carried out—sketches, mapping, among others.”⁵ Our RbD framework was structured in an iterative cycle of (a) geometric modeling, (b) physical prototyping at half-scale, (c) parametric modeling and structural analysis, and (d) full-scale prototyping (Figure 3). As discussed in more detail in the Conclusions, it is important to emphasize that this was not a linear process, but a cyclic one in which the development of each stage fed back important insights into the rest of the process. Initial geometric models were developed in Rhinoceros 3D; parametric models were built in Grasshopper (a visual programming language and environment that runs within the Rhinoceros); Finite Element Analyses were conducted using ANSYS, an engineering simulation software. The assembly sequence was implemented in GHpython, the Python interpreter component for Grasshopper. In the early stages of the research components were fabricated using a CNC router, but the team soon leaned towards a 70-Watt commercial laser cutter for both half-scale and full-scale fabrication.

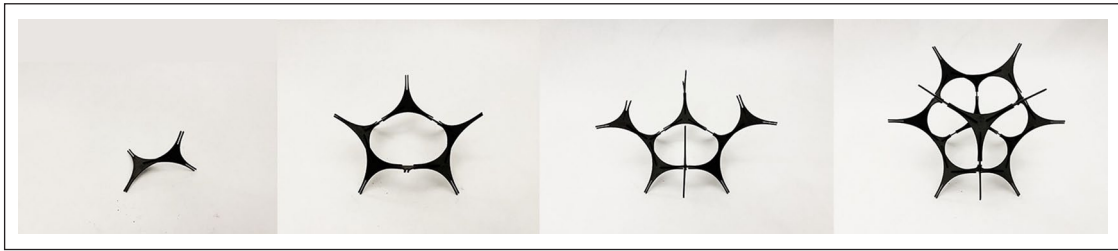


Figure 2. The Melnikov press-fit kit assembly sequence.¹²

The Weaire-Phelan structure is an aperiodic polyhedral structure developed in 1993 by Denis Weaire and Robert Phelan, two physicists working at Trinity College, Dublin.¹³ Weaire and Phelan discovered that by minimizing the surface between different polyhedra this structure can fill space more efficiently than previously thought possible. The structure comprises two kinds of cells of equal volume: an irregular pentagonal dodecahedron and a tetrakaidekahedron, a polyhedron comprising two hexagonal and 12 pentagonal faces (Figure 4). We confronted several challenges. First, the design of interlocking elements allowing for non-orthogonal connections. Traditional flat-cut assemblies require that two elements join at right angles at their dihedral joint. This constraint significantly limits the generalizability of flat-cut construction kits, especially in its application for polyhedral spatial structures such as Weaire-Phelan, where connections between elements are mostly non-orthogonal. A second challenge concerns the planning of a viable assembly sequence. While parts need to be assembled without collision, the structure needs to be free from force concentration and maintain equilibrium. To address these two challenges we introduce an adaptive jointing system to accommodate non-orthogonal connections and a graph-based model to generate a sequence of motions to assemble elements into the targeted spatial structure. The details are unfolded below in the following order: (a) Kit design, (b) Computation of adaptive joints, and (c) Planning of assembly, and (d) Full-scale prototype.

Kit design

In order to converge into a general geometric strategy to realize Weaire-Phelan structures, a series of prototypes were developed in 1/32", 1/16", and 1/8" thick cardboard. The resulting fabrication kit comprises nine elements which are parametrically generated to adapt to the conditions of deformation required at each vertex of the structure (Figure 5). The shape of each element is defined by the number of friction fit connectors and rational Bezier curves connecting adjacent connectors. By changing the weight of curves within the range from zero to one, the element's bounding shape can be adjusted between ellipse, parabola and hyperbola. Finally, a NURBS surface was defined by the boundary curve.

As a construction method, interlocking assemblies allow for a wide range of materials, scales, and fabrication techniques. In this paper we focus on assemblies composed of planar, CNC-fabricated interlocking pieces. Our fabrication kit for the Weaire-Phelan structure comprises nine elements, which are parametrically generated to adapt to the conditions of deformation required at each vertex of the structure (Figure 5). The shape of components is defined by the number of friction fit connectors and rational Bezier curves connecting adjacent connectors. By changing the weight of curves within the range from zero to one, the component's bounding shape can be adjusted between ellipse, parabola and hyperbola. A NURBS surface was defined by the boundary curve. To test the approach, prototypes were developed in 1/32", 1/16", and 1/8" thick cardboard in preparation for a final installation built with 1/4" thick Baltic birch plywood.

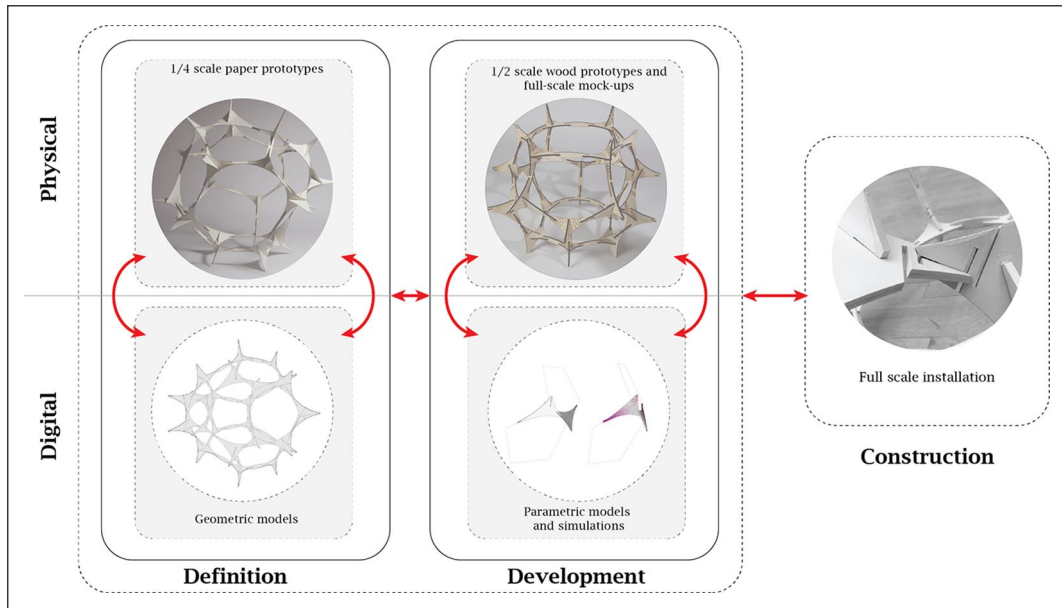


Figure 3. Framework of the research by design methodology.

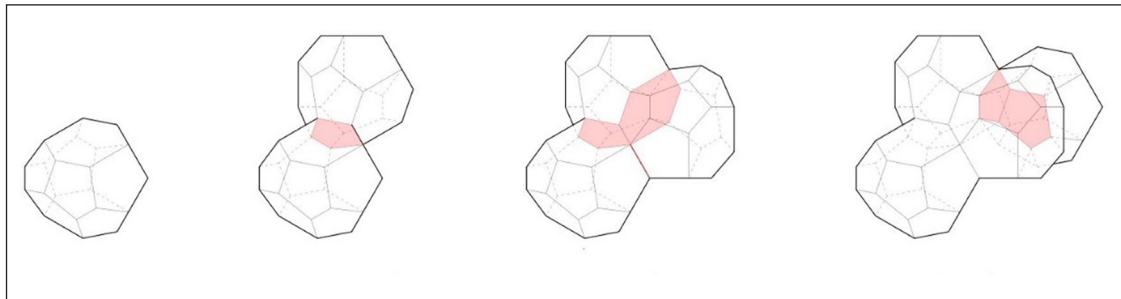


Figure 4. Aggregation method of the Weaire-Phelan structure.

Computation of adaptive joints

The design of the kit for resolving the Weaire-Phelan structure for fabrication showed that each joint had to adapt to local geometric conditions at each vertex of the structure. Only by creating variable bending conditions the kit could ensure that the elements joined at right angles, increasing the quality of the assembly. We calculated these conditions in the parametric model, developed in Grasshopper, by tracking angular deviations between assembly planes at each vertex. Through prototyping iteratively several assemblies, we related the angular deviation to an adaptive kerf pattern that causes the element to bend in response to each joint's specific conditions. Along with a variable for the material's thickness the model generates a notch that suits the assembly's role as either a "locked" or "free" element. We abstracted two connected components as two nodes V_i and V_j . The slits between components are represented as an edge E_{ij} connecting two nodes in the graph. The angle α between two connected components can be calculated by

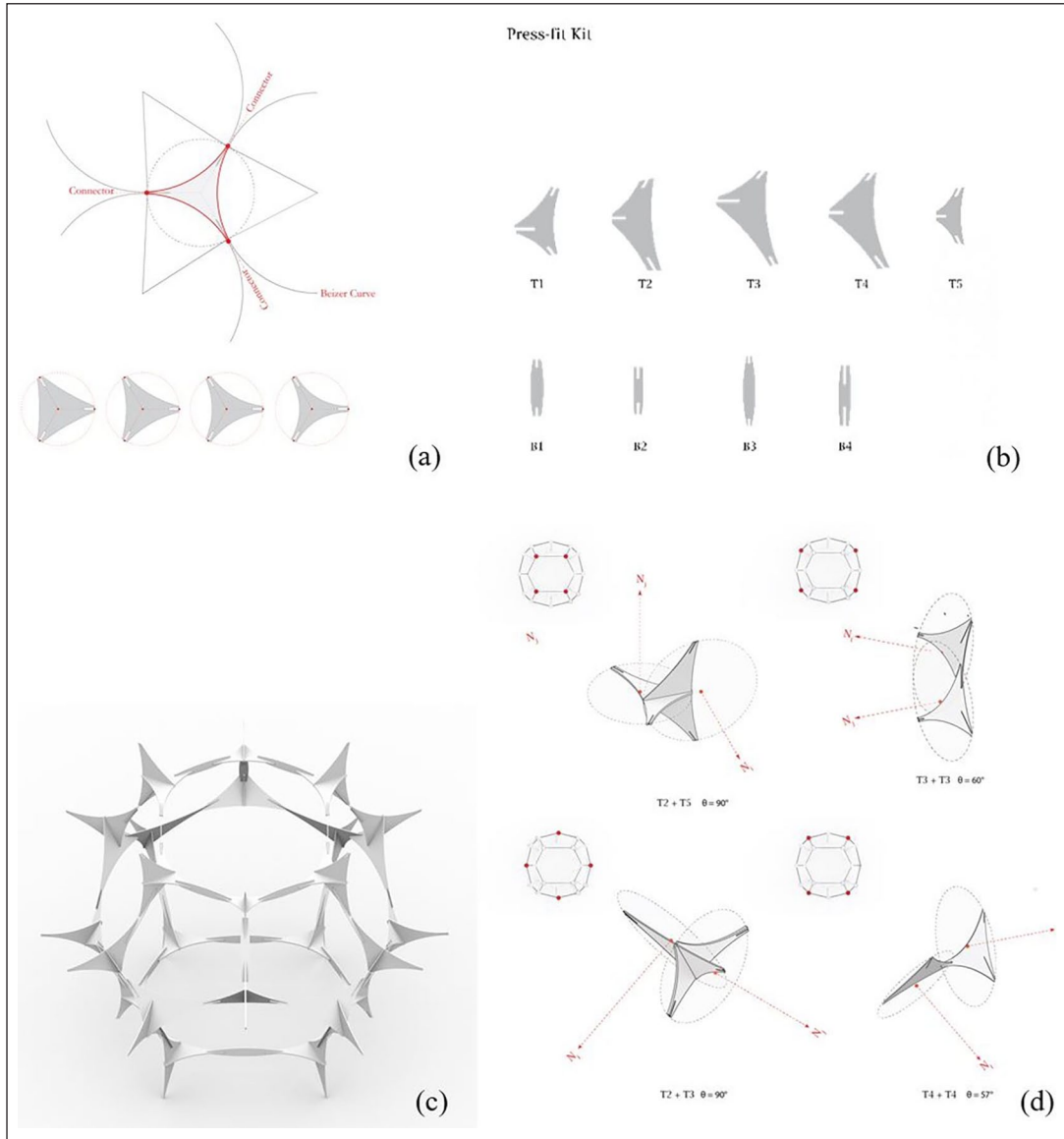


Figure 5. (a) The press-fit components defined by the number of fit connectors and Bezier profile curves, (b) The press-fit kit designed for the construction of the Weaire-Phelan structure, (c) The Weaire-Phelan structure assembled from the press-fit kit, and (d) Configuration of components at identical vertices of the Weaire-Phelan structure.

$$\theta = \arccos \frac{N_i \cdot N_j}{|N_i \cdot N_j|} \quad (1)$$

where N_i and N_j are orthonormal vector of two components (Figure 6). For connected components, nodes are categorized into two types, free or locked. Components denoted as locked have a tight slit, which equals to

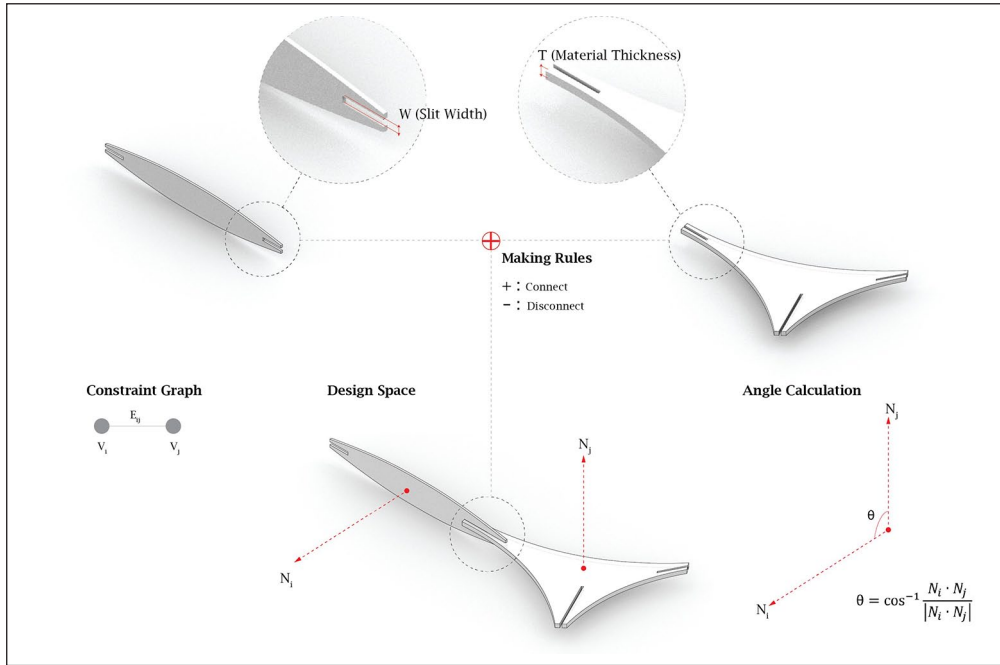


Figure 6. Components V_i and V_j connected by the press-fit joint.

the thickness of material. The locked node is constrained to be locally rigid which cannot slide or rotate. By slightly widening the notch, two connected components can rotate along the axis with bounded angle range.

An important aspect of the adaptive joint is that the planes forming assemblies at many vertices in the structure are not orthogonal. This condition can reduce the effectiveness of friction-fit joints. In order to approximate an orthogonal condition, we implemented an adaptive kerf bending function into our joint design. The parameters controlling kerf shape are material thickness (T), number of columns of torsion links (N), notch width (W), notch length (L) and kerf spacing (D). We used finite element analysis to validate the bending limit of the connector in according to the parameters assigned (Figure 7). In addition, FEM simulations were used to estimate the behavior of the structure under typical geometric and physical conditions. As we can see in Figure 6, two press-fit components connected orthogonally need to twist 23.77° along their shared main axis to resolve a specific the Weaire-Phelan structure vertex. The twisting is enabled by the deformation of adaptive joints (Figure 8).

Assembly

Through iterative prototyping we discovered that to maintain the structural rigidity and avoid self-intersections, two constraints needed to be maintained. We illustrate these with an un-directed graph representing the structure and guide assembly (Figure 9). Each node in the graph stands for a press-fit component. A pair of directed edges corresponds to the connection between components in contact. Theta denotes the goal relative angle between components after assembly. To determine whether the structure can be assembled, the inverse way is to find a sequential order to disassemble the structure into separate components.¹⁴ If the order exists, the component can be assembled into the target structure in a reverse manner. Constraints to avoid intersection and structural instability need to be met over the search for a disassembly sequence. In this particular work, the

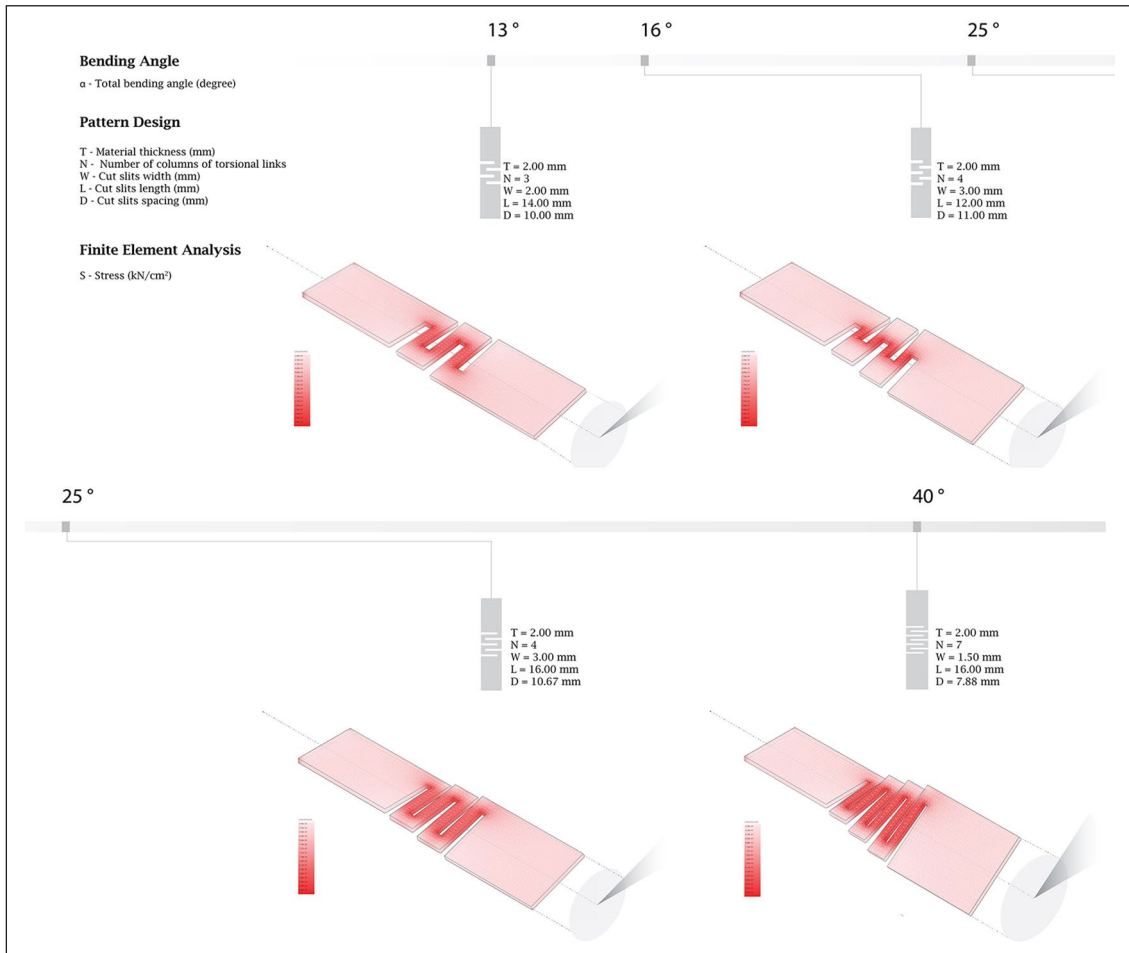


Figure 7. Kerf pattern created based on the target torsion angle.

constraints are of two types: motion and graph. Motion constraints limit motions of two connected pieces to translations along a shared axis with opposite directions. The shared axis is defined by the vector perpendicular to the plane defined by the orthonormal vector to the surface of two connected components.

Graph constraints ensure that components can be separated or assembled without a collision. A graph is separable if it can be partitioned into more subsets that can be further partitioned or it only contains one component. A graph A can be partitioned into two sub-graphs when two requirements are met:

- $B_1 \cup B_2 = A$ and $B_1 \cap B_2 = \emptyset$
- A pair of parallel edges, or a pair of edges the angle between which is within the range of bending limit of the kerf pattern, exists in both sets.

A structure consisting of many interlocked components can be analyzed in a hierarchical fashion: the large aggregation can be decomposed into several smaller interlocked aggregations. As can be seen in the (Figure 8), the set A can be partitioned into two subsets B_1 and B_2 by cutting two parallel edges. However, A

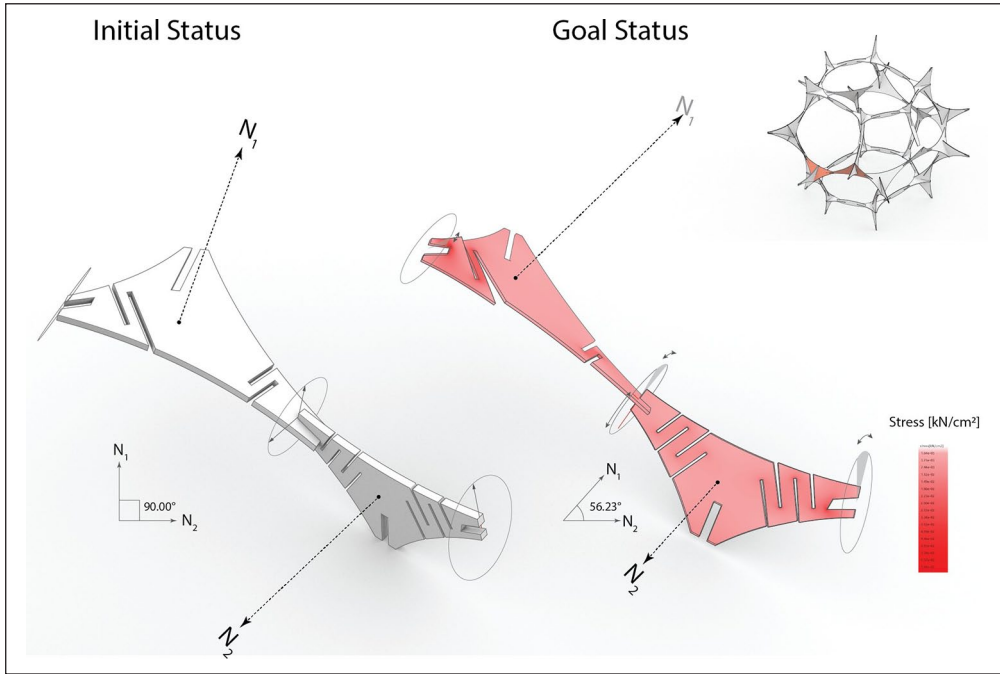


Figure 8. The twisting of two connected components enabled by adaptive joints.

can't be partitioned into sub-graph C_1 and C_2 since there is no parallel edges shared by C_1 and C_2 . The angle θ defined by two components which are to be disconnected is larger than the bending angle $\alpha_1 + \alpha_2$ that can be afforded by their kerf pattern. The disassembly of graph A into sub-graph C_1 and C_2 can result in a structural failure of the components with kerf pattern. Similarly, B_1 and B_2 can be assembled into A , but C_1 and C_2 are not able to be assembled without self-intersection or structural failures. By partitioning the separable graph recursively, we can get the sequence of disassembling the structure, the reverse of the order is an instruction for assembly (Figure 10).

Results: full scale, flat-cut Weaire-Phelan structure

The previous sections discussed how through parametric models and small-scale prototypes we explored the variations of element design, and through finite element simulations we explored its performance. This section discusses the preparation of a full-scale prototype based on the Weaire-Phelan structure to test the approach at an architectural scale.

As the prototypes and simulations discussed above showed, in our prototype Weaire-Phelan structure not all connections are orthogonal. Figure 8 shows the dihedral angle between two connected elements is 55.23° . The non-orthogonal connection incorporates the kerfing pattern defined by parameters informed by the simulation results (Figure 7). The kerfing pattern ($T = 2.00$ mm, $N = 4$, $W = 3.00$ mm, $L = 14.00$ mm, $D = 10.67$ mm) and the kerfing pattern ($T = 2.00$ mm, $N = 3$, $W = 2.00$ mm, $L = 16.00$ mm, $D = 10.67$ mm) can afford the rotation of 38.00° along the axis without failures. Thus, in total, the adaptive joints can afford the connection with the node angle ranging from 52.00° to 128.00° . Considering the targeted dihedral angle is 55.23° , the tolerance in the dihedral angle between the connected elements is 3.23° .

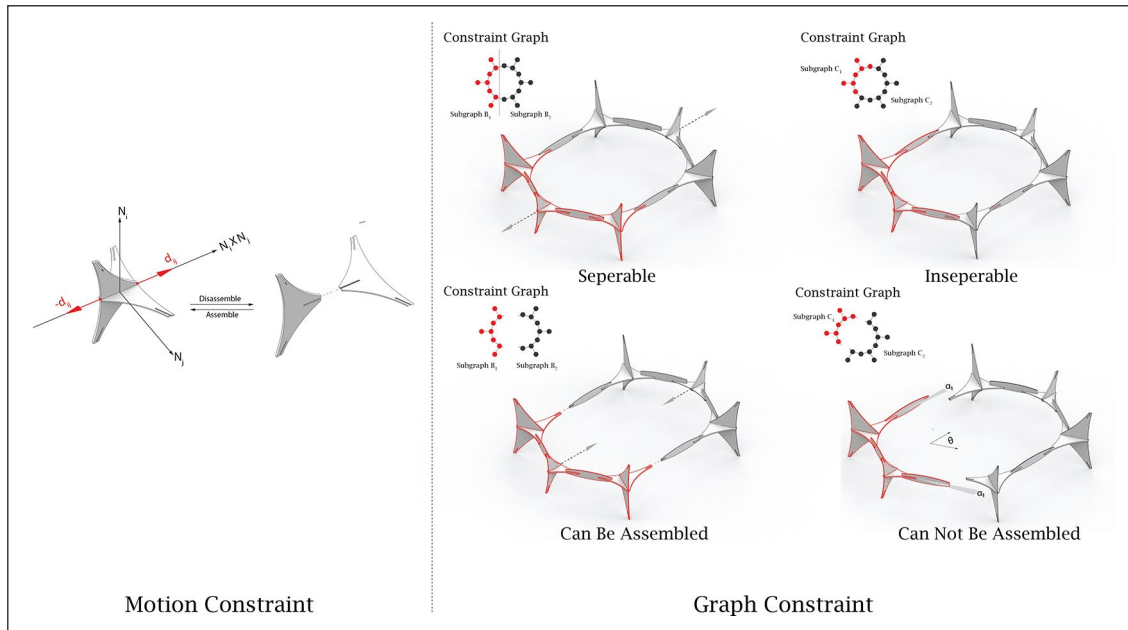


Figure 9. Mechanical and 4-D assembly constraints to avoid self-intersection.

The production was organized in individual polyhedral, or “units,” built with the nine elements described in the previous section. Each unit was produced in under approximately 2 h by cutting the individual elements in 1" thick Baltic birch plywood with a 50-Watt laser-cutter, assembled, and treated with spray paint and gesso (Figure 11). The resulting unit appears light and smooth, focusing attention on the geometric and structural aspects of the prototype rather than on the material's textures. On-site assembly was conducted in a bottom-up fashion: setting a base layer and building up until we reached five layers. The structure comprises 15 individual polyhedra of approximately one cubic meter in volume (Figure 12). Each polyhedron is composed of three rings, and total of 74 individual pieces in nine types. To avoid collision and maintain equilibrium over the assembly process, we used the motion sequence generated from the graph-based model illustrated in the previous section. Building tightly fit and robust individual polyhedra and progressively assembling them “into” the structure helped correct errors and deformations introduced by gravity and differentials in material thickness. However, several pieces had to be recut and the model adjusted, suggesting that material variation affects the reliability of the adaptive joint.

The prototype's overall shape resulted from in-situ decisions during the assembly process. For example, noticing that adding units on the fourth and fifth layers noticeably increased the pressure on the lower assemblies, we restricted the height to avoid over-stressing the structure. When two polyhedra are stacked, one overlaps the other by half its height. Thus, five layers were enough to accomplish our desired height of six feet and six inches—taller than a human body. Assembling the final installation took a team of two people a total of 15 h. The structure was on display during a period of 4 days in the context of a school-wide exhibition of work.

Conclusion and next steps

Architectural space frames today are constructed using one of three major systems: bar and node systems, plate structures, and combined bar and plate structures. Following a Research by Design methodology, in this paper we have documented the successful construction of an architecture-scale space frame based on the

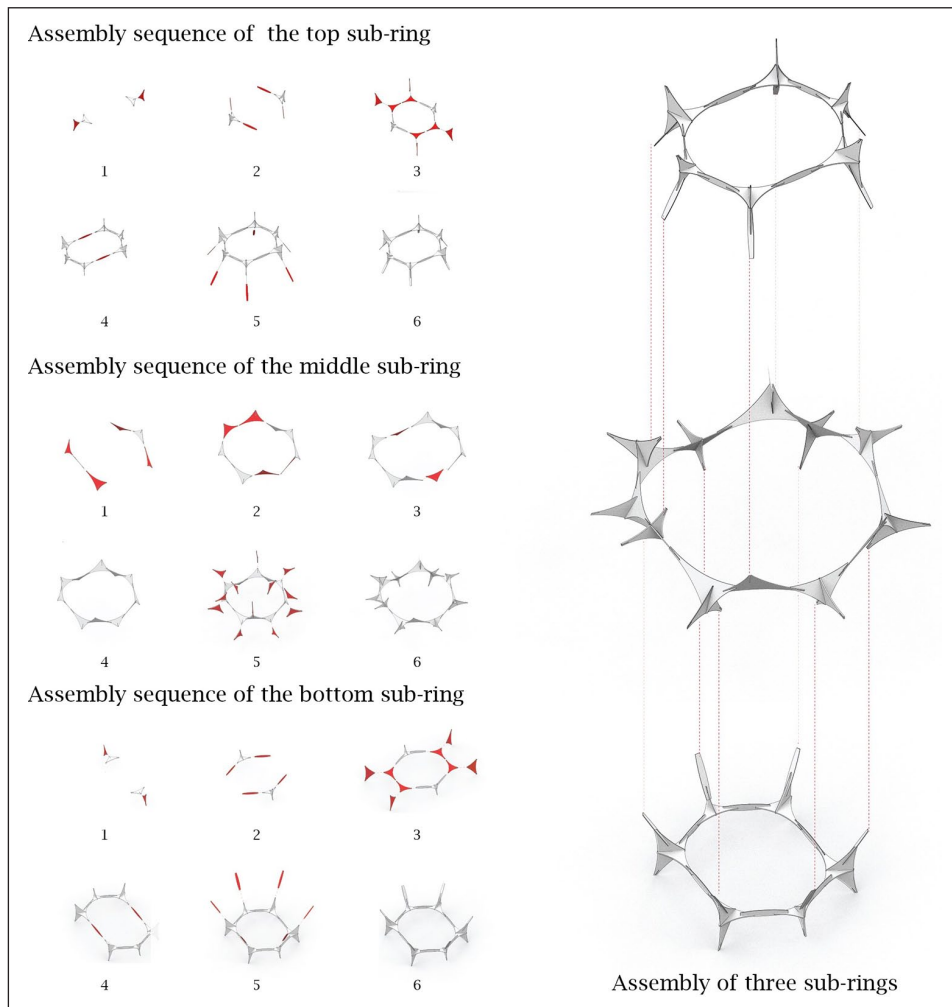


Figure 10. Assembly sequence of a Weaire-Phelan structure polyhedron.

Weaire-Phelan structure—a geometric structure that approximates the geometry of foam—based on parametrically adaptive elements that adjust to local geometric and mechanical conditions; is made entirely from flat-cut elements that interlock without the need for nodes, fasteners, cement, or glue; and is suited for low-cost rapid prototyping and manual assembly.

Of particular interest here is the combination of parametric and modular logics. The structure's parametrically defined elements bend in response to local geometric conditions. Through iterative prototyping and simulation a variable kerfing pattern was defined to help ensure the integrity of the assembly. Addressing issues encountered in an earlier study,¹² we found that an adaptive kerfing pattern that is responsive to local conditions can facilitate out-of-plane torsion for non-orthogonal, friction-fit assemblies, which are essential for flat-cut space structures. The result is a modular system that nevertheless adapts individual elements to material and mechanical constraints. In addition, the prototype showed how processes of bottom-up design exploration can be tightly coupled with their tectonic resolution. For example, the modular nature of the system makes it possible to achieve multiple geometric configurations (see, e.g. Figure 4). However, it is

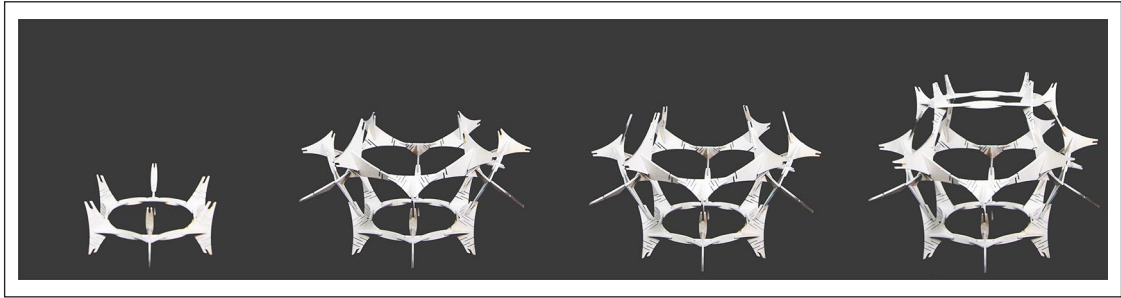


Figure 11. Assembly sequence for a prototype of a single Weaire-Phelan polyhedron.

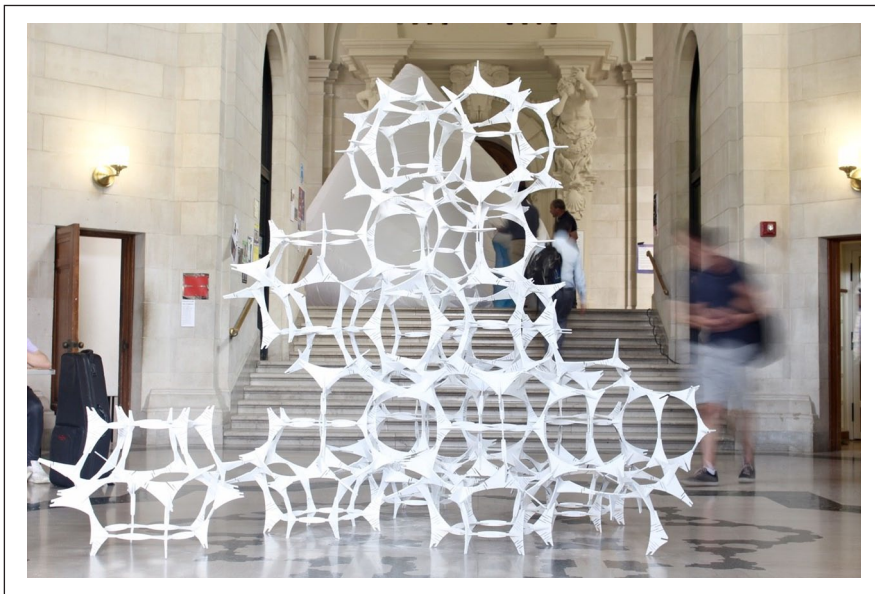


Figure 12. The full scale flat-cut Weaire-Phelan structure installation (Photo by Chitika Vasudeva).

important to observe that a characteristic of Research by Design methodology is that results are often highly specific to the particular material and geometric strategies employed in the research. In our case, these relate to the prototype's materials, assembly method, and scale. For example, in building the full-scale prototype we learned that the accumulated stress in the bottom modules limits the vertical growth of the installation to around 7 ft. Accordingly, ascertaining the scalability and generality of the approach to other scales and geometric configurations requires further investigation.

It is also important to emphasize that the design to analysis and fabrication framework discussed (see Method) was not linear, but cyclical, and that the different stages informed each other in ways that were not fully prescribed at the onset of the investigation (see Figure 3). As the research progressed, for example, paper models made visually evident the need for out-of-plane torsion in the structure's components, while parametric models were instrumental to measure angular deviation at each node of the assembly—thus providing a template for the adaptive kerfing pattern. The physical prototypes and full-scale mock-ups helped the team to probe this logic, and were crucial to refine the design of the elements and to calibrate the adaptive

joint design. As we learned about the material's tolerances through half-scale prototypes and full-scale mock-ups, the kerfing pattern was adjusted to account for the material's behavior. Similarly, as the team discovered issues with the assembly sequence in the physical construction of the prototypes, the sequence was adjusted to better account for collisions and over-constrained nodes. Through several iterations developed over the course of one semester, the component and adaptive joint designs encoded with increased precision the behavior of the material assemblies, until the team reached a level of confidence in the process that invited the construction of the full-scale installation.

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