

A Minimal Making Grammar for 3-D Structures

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Abstract. The design of systems of interlocking, digitally-fabricated parts has been a subject of interest for computational design researchers for some time. A discrete language of elements and a set of rules for their assembly, some of these efforts have shown, can circumscribe an open-ended space for design and material exploration. Taking cues from Shape Grammars, a foundational theory and praxis of generative design, and Making Grammars, this paper contributes to these efforts by introducing a minimal making grammar which innovatively comprises a single, easy to manufacture element. Adapting traditional mortise-and-tenon joinery techniques to contemporary digital manufacturing methods, this minimal making grammar allows for the design and construction of light, three-dimensional structures without the need for nails, fasteners, or adhesives. Describing the grammar through procedural descriptions, material studies, structural analyses, and the evaluation of four objects derived from it, the paper characterizes this grammar, its design space and structural behaviour highlighting how, despite its specificational simplicity, it allows for a great deal of spatial and visual complexity.

Keywords. Computational design, generative fabrication, making grammars, shape grammars, joinery system, computer-aided manufacturing

1. Introduction

The design of systems of interlocking parts for digital fabrication has been a subject of interest in computational design for some time. Researchers have created, for example, generative fabrication grammars (Cardoso Llach and Sass, 2008; Liu et al, 2021); systems to automate or optimise design (Barda et al., 2023; Wu et al., 2019; Santos et al., 2018), support creative exploration (Zhou et al., 2023), or bridge traditional crafts and computational strategies (Dinçer et al, 2024; Noel, 2020). Drawing from Shape Grammars—a foundational theory and praxis of generative design (Stiny and Gips, 1971)—and Making Grammars (Knight and Stiny 2015), a portion of these efforts has been aimed at exploratory systems where a language of elements, and a set of rules for their physical articulation, define an open-ended design-fabrication, or making, space.

This paper contributes to these efforts by introducing a minimal making grammar which comprises a single, easy to manufacture, element. Innovatively adapting traditional mortise-and-tenon joinery techniques to contemporary digital manufacturing methods, the grammar allows for the design and construction of three-dimensional structures without the need for nails, fasteners, or adhesives. This grammar, we will show, is specificationally simple while allowing for a great deal of spatial and visual complexity.

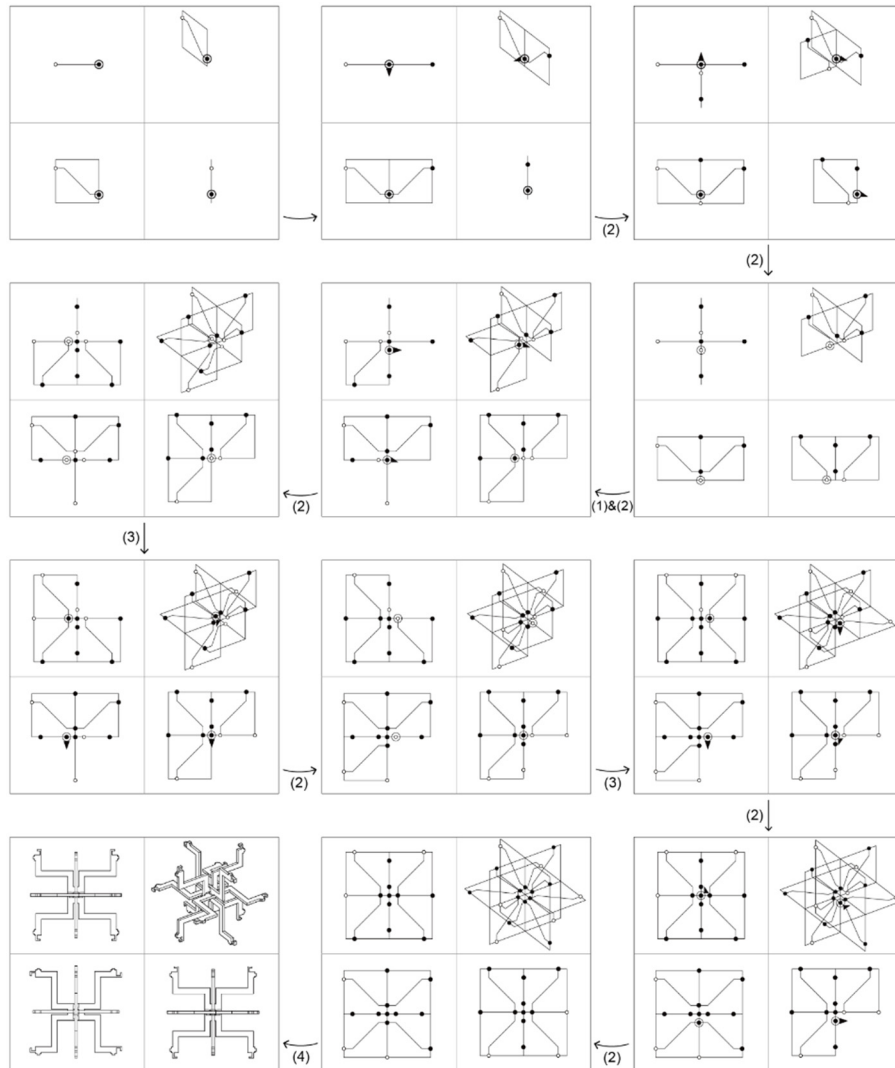


Figure 1. Derivation of a complete interlocking joinery according to Configurational rules.

2. A minimal making grammar

Adapting the classical Shape Grammar formalism (Stiny, 1976; 1977), the minimal making grammar is specified with the 5-tuple $E = (Vt, Vm, Rc, Rt, I)$ where E = A set of elements, Vt = Terminals, Vm = Markers, R = Rules of element transitions from left to right, I = Initial element, consisting of elements in Vt, Vm . Vt comprises a single “seed” element featuring two hook-tenons. The figure is directional, with its outward hook marked as a white dot, and its inward hook is marked as a black dot. Configurational rules (Rc) specify the generation of 3-D structures, and tectonic rules (Rt) specify the physical articulation of the elements. As shown in Figure 2, the circle marker in set Vm indicates the current endpoint for interlocking, positioned either at the white or black dots as shown in I .

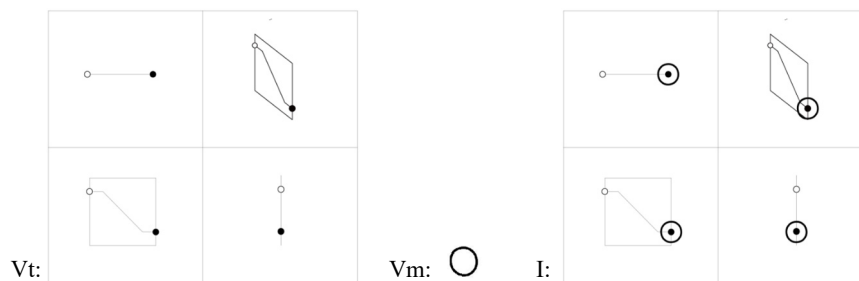


Figure 2. Vt: A representation of seed elements from four views (clockwise from top left, Top, Perspective, Right, Front). Vm: marker. I: the marker shape in Vm over the element in Vt.

2.1 CONFIGURATION RULES

Configurational rules (Rc) specify the generation of three-dimensional structures (Fig. 2) where Rc (1-3) are transformation rules, and Rc (4-6) are substitution rules. Rc (1) depicts a black dot in the marker connected to a white dot of the other element to expand in the same plane alignment, as shown in Figure 3.

The additive element on the right side of the rule needs a 180-degree rotation around the x-axis and a one-unit translation relative to its initial position on the left. Rc (3) illustrates an additive perpendicular transformation. The element first rotates 90-degrees around the y-axis from one of the initial elements, followed by a 90-degree rotation around the x-axis and a one-unit translation along the z-axis. This perpendicular transformation rule applies only when the condition involves more than three elements in the right image, as in cases Rc (2-3). Rc (3) can extend in two directions, back and forth, marked by an arrow in the views as needed. Configuration rules (4-6) illustrate the substitution of 3-D shapes into a specific seed element. The derivation of an interlocking module with twelve units of a “Z” type seed element, substituted by Rc (6) shown at the bottom left is showcased in Figure 1. The final step shows how the opposite dot at each of the twelve elements has the potential to initiate another interlocking system conforming to the configuration rules. The dots’ positions align with the vertices of a cuboctahedron, a combination of four vertices of three squares in the x, y, and z planes.

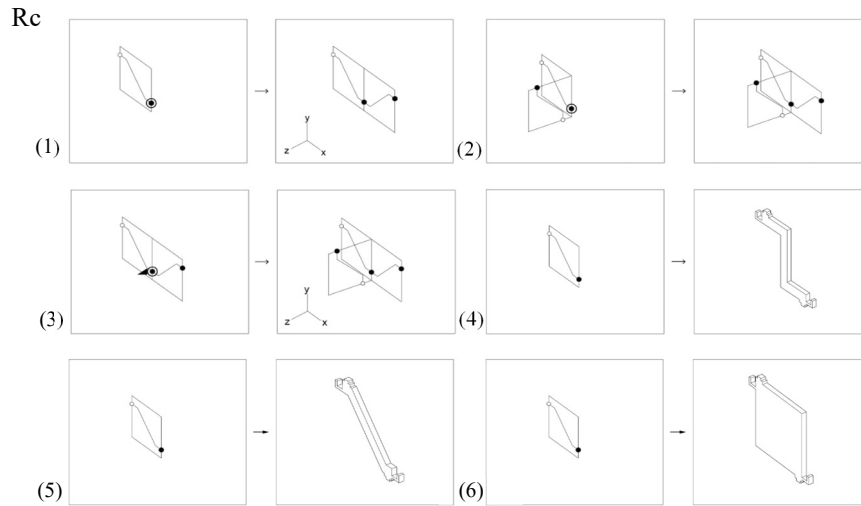


Figure 3. Configuration rules (Rc)

2.2 TECTONIC RULES

The physical articulation of elements is specified in tectonic rules (Rt), illustrated in Figure 4. Rt (1) specifies how two elements align in the same plane making a mortise with two fastened hooks, as depicted in Rc (1-2). Rt (2) illustrates how a tenon in the additive element fits vertically into the mortise, supporting the configuration rule (3).

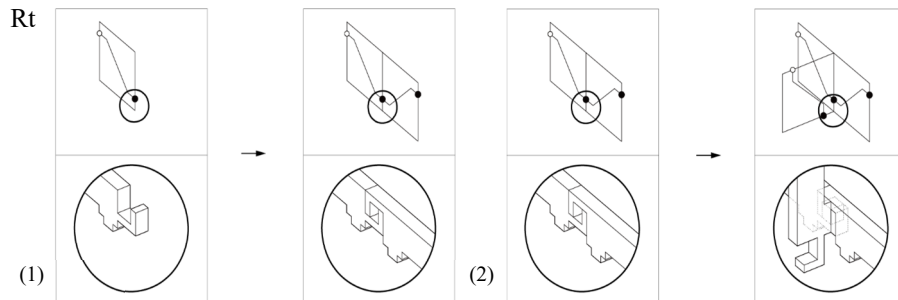


Figure 4. Tectonic rules (Rt). A black dot indicates an inward hook or mortise with two hooks.

The tenon's insertion in the mortise creates static friction on both contacting surfaces, securing the joint. The derivations in Figure 5 (a-d) show how an additional seed element fits into the joinery node based on the set Rt. All derivations adhere to Rc. While a single joinery node, or module, can accommodate up to twelve edges, a minimum of three is required for an effective hook-tenon system.

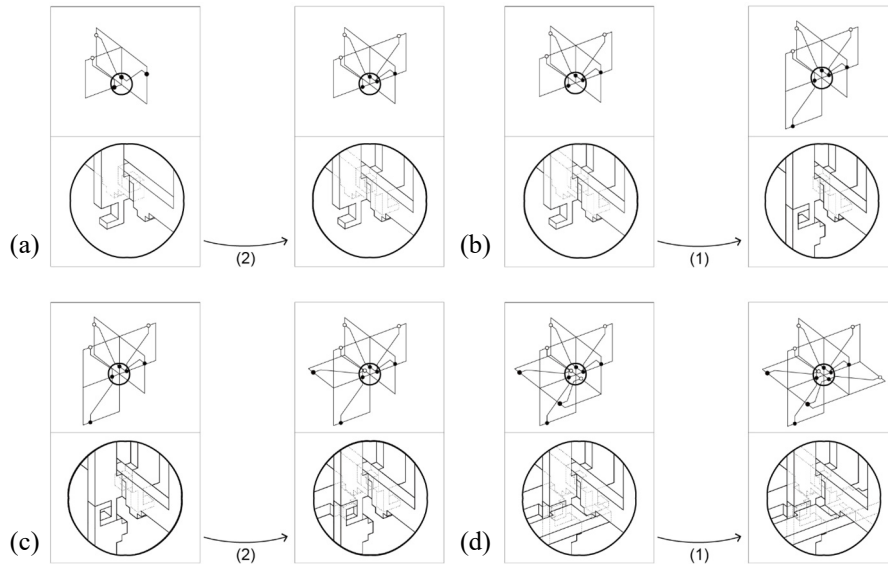


Figure 5. Four cases of generations in a joinery node following the Rt. (For each drawing(a)-(d): Top - Perspective view; Bottom- Zoomed-in view of the circle area)

2.3 THE ELEMENT IN DETAIL

Our minimal making grammar draws on traditional, friction-based wood joinery principles, taking advantage of the precision and economy of digital fabrication methods. Pressure, generated by strain, produces static friction in a hook-tenon joint. For this, the mortise width w is slightly smaller than the shape depth. The dimension of the mortise width w correlates to the hook length ($b + w + b$, when b is the hook width). As described in section 2.4, aspects of the fabrication process such as material removal due to laser burning, and the need for strain to achieve a tight joint, lead to further tolerance adjustments during prototyping.

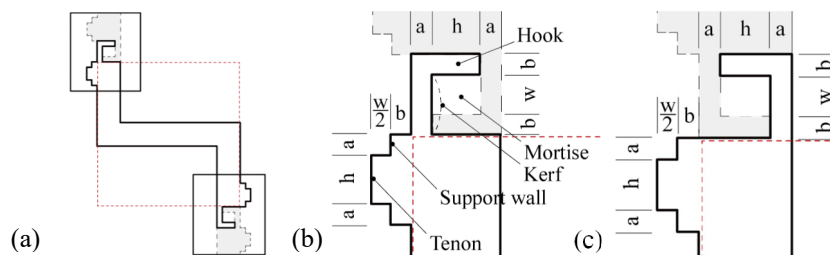


Figure 6. (a) "Z" type seed shape, (b) Inward hook with tenon, (c) Outward hook with tenon.

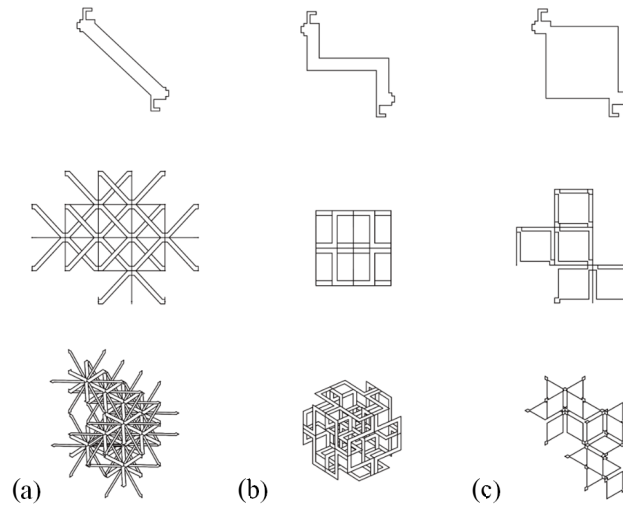


Figure 7. Three versions of the seed element. Forms within a given frame define the structure's visual expression.

2.4 REFINING THE GRAMMAR

Following the development of a parametric model in Rhino Python for Rhinoceros 3D, a series of physical tests were made to refine material and geometric aspects of the grammar. This required patience and iterative prototyping. After testing different plywood thicknesses, 6mm plywood was chosen for prototyping because of practical concerns related to its rigidity and the fact that it could be cut using our lab's laser cutter (Rabbit 60W), thus allowing for quick iteration. Accuracy is determined by the combination of the laser cutter settings and wood type, the lens's Z-axis position, power, speed and the number of passes. In addition, the material burned by the laser needs to be considered. Overall, the mortise width in the drawing is set to be 0.5 - 0.6 mm smaller than that of the physical model. The difference in mortise width and element depth, or the tenon width, is less than 0.1mm.

The bump design added to mortises facilitates tolerances for interlocking parts, and the edges of the tenon were additionally refined with a fillet (Fig. 8a). Further, support walls around the tenon were designed to secure the static friction force while allowing sufficient clearance during fabrication. In addition, we found that assemblies with less than three elements require, for their stability, of special elements to replace the tenon or the hook (see Figure 8b). The U type, for example, substitutes a mortise when the tenons of two elements touch each other. The I type replaces a hook position when another hook and a tenon meet perpendicularly, as shown in Figure 3, Rc (2). Tenon type replaces a tenon position when the hooks accommodate a mortise, as shown in Figure 3, Rc (3).

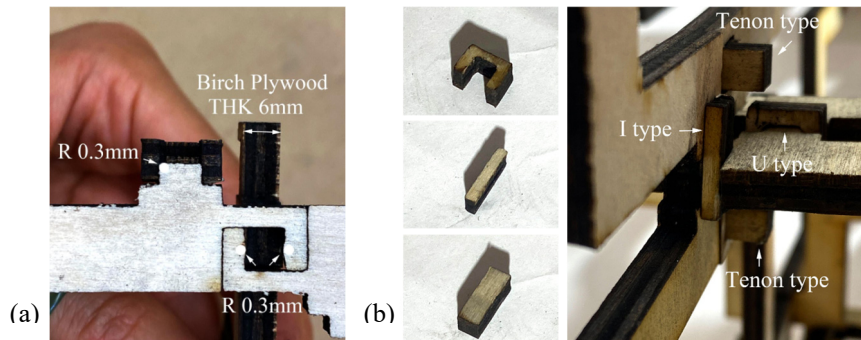


Figure 8. (a) Bump design, (b) Extra end joints (from the top: U type, I type, Tenon type)

3. Results

3.1 FOUR OBJECTS

Using the minimal making grammar, we made four different objects in birch plywood (See Figure 9). White wood paint was applied after assembly to improve visual articulation. For all objects, the seed element frame dimensions were 91.6mm x 103.2mm. Different seed elements were used, including some illustrated in Figure 7, to explore their effect on the visual and spatial qualities of objects created with the grammar. In Figures 9 and 10, Object 1 connects with a straight line (Type A), Object 2 connects in a circular shape (Type B), and Objects 3 and 4 connect in a Z shape (Type C). As seen in these figures, the spatial and perceptual qualities of each of these objects are significantly different, showing that the minimal grammar can yield a wide range of visual and spatial results. However, they are tectonically equivalent.

3.2 STRUCTURAL TRAITS

Manual and visual evaluations of the four objects showed that once assembled they are robust. Two tests conducted using an Instron machine confirmed this, showing that the compressive strength tests of a twelve-piece joint averaged 3.2kN. The static friction force of a single hook-tenon joinery, for comparison, was only 30N. These tests conformed with trends observed in preliminary simulation results. Using Solidworks, the maximum static friction was achieved at a compressive stress of 265.4MPa, calculated manually by multiplying the strain ΔL and the Elastic Modulus (2.9GPa) while applying a distribution load of 50N. Due to the inconsistencies in ΔL caused by uneven contact surface and measurement errors of the contact surface, further calibration is required to obtain more accurate results. We further observed that once the twelve-piece joint is completed, the role of static friction in sustaining the structure decreases. The structure is now primarily held together by resistive forces by the mechanical locking mechanism, like a snap-fit joint.

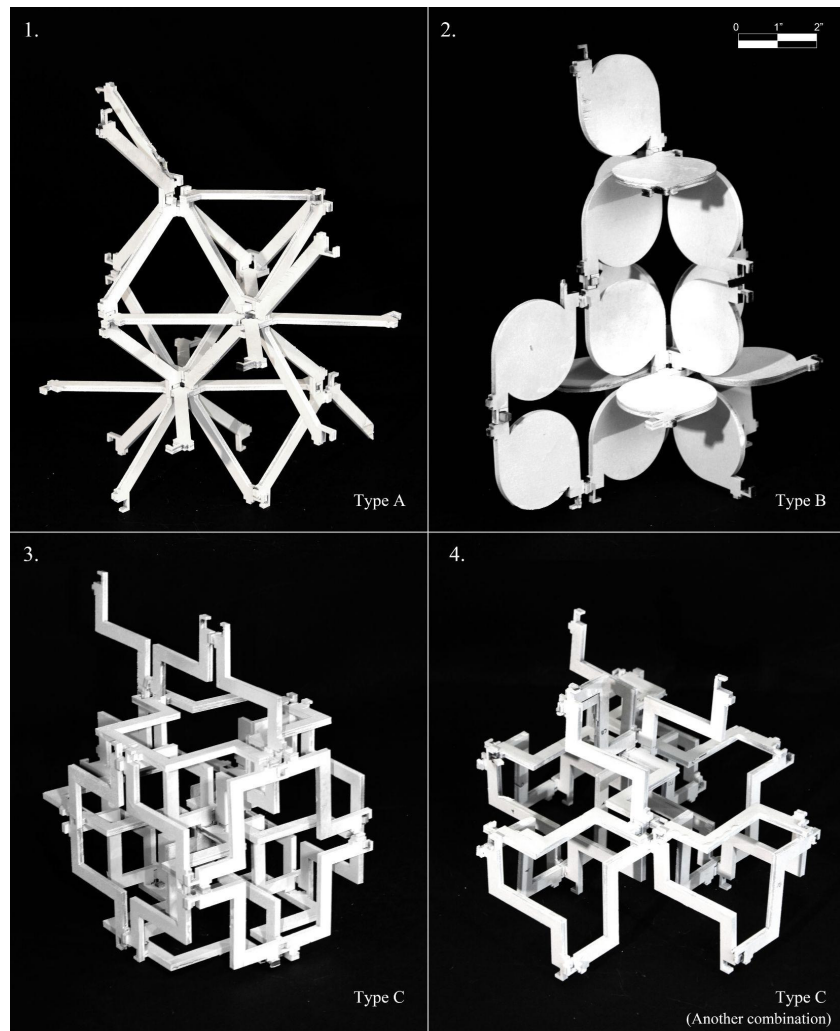


Figure 9. Four different configurations were developed using three types of seed elements.

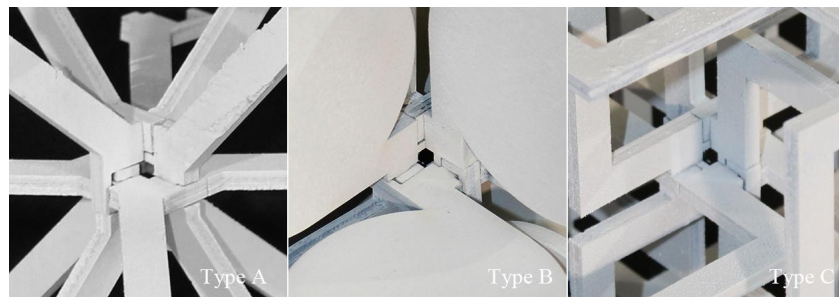


Figure 10. Detailed views of three different interlocking joints composed of twelve pieces.

4. Discussion

4.1 CONTRIBUTIONS

A novel minimal making grammar comprising a single element, and a series of rules for its articulation into physical 3-D structures, was introduced. Four objects made following this grammar were presented and evaluated. The results show that the grammar can be used to make objects which are structurally sound and spatially complex (Figure 11).

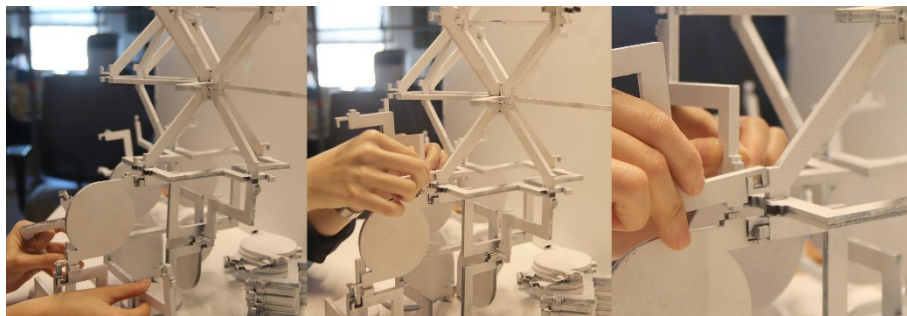


Figure 11. Making process using three types of elements

4.2 LIMITATIONS

The configurational and tectonic rules introduced in sections 2.1 and 2.2 adequately describe the creation of robust MMG assemblies. However, in making the objects we found that the derivation sequence in Figure 1 is not always easy during assembly because of the need for clearance to avoid interfering with an element's hook, and the need to exert force to push a tenon in a precise direction. For this reason, it is important to consider the derivations presented above as a starting point, or useful resource, guiding the assembly rather than as a fixed or invariable recipe.

Manual assembly requires a skilled combination of force and precision. In making the objects, we noticed this embodied skill progressively developing. In future iterations, larger elements will allow for larger clearance, simplifying assembly. Additionally, variations in laser power or in the material can create irregularities and cause assemblies to be loose or overly tight. This can be mitigated by carefully monitoring the machine's settings and by accounting for a ~10% margin of flawed pieces.

4.3 NEXT STEPS

The potential of the minimal making grammar to support the creation of furniture, sculptural elements, and 3-D architectural installations will be explored. Additional structural analyses in simulations and in the lab will be conducted, and a computational tool for 3-D form-finding will be implemented. These will, respectively, further characterise the material and structural aspects of the grammar, and open up paths towards its creative use in the making of novel objects and spaces.

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